18. V. V. Levdanskii, V. G. Leitsina, and N. V. Pavlyukevich, "Toward a kinetic transport theory for heterogeneous chemical reactions in highly dispersed porous bodies," Inzh.-Fiz. Zh., 28, No. 4, 658-660 (1975).
19. V. V. Levdanskii, "Kinetic theory of rarefied gas flow in highly dispersed porous bodies," in: Mathematical and Physical Questions in Heat and Mass Transport [in Russian], Inst. Tepl. Mass. Obm., Minsk (1973), pp. 3-8.
20. M. Asaeda, S. Yoneda, and R. Toei, "Flow of rarefied gases through packed beds of particles," J. Chem. Eng. Jpn. 7, No. 2, 92-98 (1974).
21. K. Cherchinianin, Theory and Application of Boltzmann's Equation [Russian translation], Mir, Moscow (1978).
22. P. Fott and G. Petrini, "Determination of transport parameters of porous catalysts from permeation measurements," Appl. Catal., 2, 367-378 (1982).
23. G. H. Neale and W. K. Nader, "Prediction of transport processes within porous media: Creeping flow relative to a fixed swarm of spherical particles," J. AIChE, 20, No. 3, 530-538 (1974).
24. A. A. Zick and G. M. Homsy, "Stokes flow through periodic arrays of spheres," J. Fluid Mech., 115, 13-26 (1982).

NONSTEADY UNIDIRECTIONAL DISCHARGE OF AN INSTANTANEOUSLY HEATED GAS WITH CONSTANT FORCED FLOW FROM A CYLINDER

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A numerical study is made of the problem of unidirectional discharge of an instantaneously heated gas from a half-open cylinder when the gas is pumped through the cylinder in the direction of the open end.

Calculation of thermohydrodynamic fields in half-open systems is of considerable interest for through-type electric-discharge quantum generators, where the uniformity of the medium has a significant effect on the working parameters of the system.

Apart from the specifics of the design of the system, the essence of the hydrodynamic process which accompanies the pressure jump in the working volume of the generator resulting from the discharge can be remodeled by the classical problem of unidirectional discharge of an instantaneously heated gas from a cylinder with one end open to the atmosphere under finite pressure [1]. It was shown in the solution of this problem that the initial pressure jump is accompanied by nonlinear oscillations of an amplitude which decreases slowly relative to the characteristic time scale. This result is in qualitative agreement with the test data obtained in [2] on a rarefaction wave tube. The slow return of uniformity in a system based on the principle of periodic-impulsive action when the only damping source is the local resistance at the open end of the cylinder (the friction against the walls has almost no effect on damping) stimulates searches for additional means of influencing the system parameters — one of which may be pumping the gas through the cylinder.

The present article studies gasdynamic processes in a cylindrical volume. One end of the cylinder is connected to the atmosphere. The gasdynamic processes are initiated by instantaneous heating of a gas in some middle section of the cylinder. We will study how the processes are affected by pumping the gas along the cylinder axis in the direction of the open end.

As usual in the gasdynamics of rapidly occuring processes, we will assume that the phenomena of viscosity, heat conduction, and external heat exchange have a slight effect on the

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 0022-0841/84/4606-0656$08.50

characteristics of the process. Thus, we will describe it in a unidimensional approximation on the basis of three differential equations: the equations of motion in the form of the Euler equations, the continuity equations for a compressible gas, and the energy equations without allowance for heat conduction. We will also use the equations of state of an ideal gas.

We take the quantities L, a_0, p_0, ρ_0, L/a_0, and T_0, respectively, as the scales of length, velocity, pressure, density, time, and temperature. We represent the complete system of equations describing the nonsteady discharge process in dimensionless form [1]:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\frac{1}{k\rho}\frac{\partial p}{\partial x}=-f(u),$$
$$\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial x}+kp\frac{\partial u}{\partial x}=0, \tag{1}$$
$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+(k-1)T\frac{\partial u}{\partial x}=0,$$
$$p=\rho T,\ f(u)=\frac{\lambda L}{8r_h}u^2.$$

This system must be augmented by initial conditions for u, p, and T and three boundary conditions to link these three functions.

The initial conditions for p and T are determined by the position and magnitude of the pressure and temperature jumps in the heating zone. The initial value for the quantity u is the flow velocity u_0. We write these conditions in the form

$$u=u_0,\ p=p_1,\ T=T_1\ \text{for}\ t=0,\ x_1\leqslant x\leqslant x_2,$$
$$u=u_0,\ p=p_0,\ T=T_0\ \text{for}\ t=0,\begin{cases}0\leqslant x<x_1,\\ x_2<x\leqslant 1.\end{cases} \tag{2}$$

At the open end the boundary condition for pressure is determined as the sum of the external pressure and the pressure loss due to the local resistance $\Delta p = k\xi\rho u^2/2$. Assuming that the local-resistance coefficient ξ during discharge into the atmosphere (as for steady discharge) is equal to unity [3], we obtain

$$p=p_0^*+k\rho\frac{u^2}{2}\ \text{for}\ t>0,\ x=1. \tag{3}$$

The boundary condition for the closed end in the absence of through motion is assigned exactly in the form of a zero velocity value u = 0 for x = 0. In the case of continuous through motion of the gas in the cylinder with a certain constant velocity in the absence of perturbations, the form of the boundary condition at the closed end will generally depend on the method of organization of this motion. Here we are examining one of the possible forms of this boundary condition. Let a volume containing a working gas under a sufficiently high constant pressure p* be connected at its end by a choke with a gasdynamic resistance ξ with a cylinder. The gas flow rate is assigned for the unperturbed state $Q = u_0F$. Assuming that the velocity in the volume ahead of the choke is equal to zero and using p and u to designate the pressure and velocity of the gas after the choke in the cylinder, we obtain the following relation on the basis of conservation of energy and momentum

$$p+\xi k\rho\frac{u^2}{2}=p^*\ \text{for}\ t>0,\ x=0. \tag{4}$$

The coefficient ξ in Eq. (4) is assumed to be constant during discharge. We determine its value from the condition of constancy of the flow rate $Q = u_0F$ for the gas in the cylinder in the unperturbed state. Replacing p, ρ, and u in (4) with their values in the unperturbed state p_0, ρ_0, $u_0 = Q/F$, we obtain

$$\xi=2(p^*-p_0)/(\rho_0u_0^2). \tag{5}$$

Thus, the problem of describing the gasdynamic processes occurring after the instantaneous heating of a gas in a certain region of a cylinder with continuous through motion of the gas in the cylinder reduces to the solution of a system of first-order hyperbolic differential equations with initial conditions (2) and boundary conditions (3), (4), which should be augmented by temperature values at boundary points. Since the system of equations and boundary conditions are nonlinear, the problem will be solved numerically. For this, in accordance with the general theory of the solution of systems of hyperbolic equiations [4], it is first of all necessary to replace system (1) by an equivalent system in invariant form (not explicitly separating into two equations of Riemann invariants) [5]

$$\frac{\partial u}{\partial t} \pm \frac{1}{k\sqrt{\rho p}} \frac{\partial p}{\partial t} + \left(u \pm \sqrt{\frac{p}{\rho}}\right)\left(\frac{\partial u}{\partial x} \pm \frac{1}{k\sqrt{\rho p}} \frac{\partial p}{\partial x}\right) = -f(u),$$
$$\frac{\partial Z}{\partial t} + u\frac{\partial Z}{\partial x} = 0, \; p = \rho T, \tag{6}$$

where

$$Z = \ln(T/p^{\frac{k-1}{k}}). \tag{7}$$

The initial and boundary conditions for system (6) are assigned in accordance with (2)-(4). The boundary value for Z at u > 0 should be assigned on the left, i.e., for x = 0, while the boundary value for Z at u < 0 should be assigned on the right, i.e., for x = 1.

In [1] the first two equations of (6) were written for the Riemann invariants

$$X, Y = u \pm \int_{p_1}^{p} \frac{dp}{k\sqrt{\rho p}},$$

which, on the basis of the equation of state and Eq. (7), can be represented in the form

$$X, Y = u \pm \int_{p_1}^{p} \exp(Z/2)\, p^{-\frac{k+1}{2k}}\, dp.$$

The integral in the invariants X and Y can be calculated if the invariant Z, proportional to the entropy, is constant. This condition is satisfied in accordance with the last equation of system (6). It should be kept in mind, however, that the quantity Z undergoes a discontinuity at the boundary of the contact surface; the values of Z on both sides of the contact surface are constant but different, and the process is nonisentropic. The invariant Z together with the contact surface is propagated in space with a characteristic velocity equal to the velocity of the medium u. The invariants X and Y are propagated with characteristic velocities $u \pm \sqrt{p/\rho}$, which with the corresponding direction of u will be greater in absolute value than the velocity of the medium u. This leads to a situation whereby the corresponding invariants will intersect the contact surfaces, where the above integral loses meaning — since the invariant Z undergoes a discontinuity. For this reason the reverse transition from the invariants X and Y to the variable u and p requires that the factor exp (Z/2), undetermined in this position, be replaced by some mean value of the factor. This unavoidably distorts the distribution of u and p near the contact surface. To avoid these difficulties, here we write a finite-difference problem directly for system (6), without a preliminary transition to invariants X and Y in the first two equations. It is understood in this case that the derivatives of p and u must be approximated by difference relations satisfying the condition of Curant, Friedrichs, and Levy [6, Sec. 24]. We introduce the notation

$$\sqrt{p/\rho} = \exp(Z/2)\, p^{(k-1)/2k} = \alpha(Z, p) = \alpha,$$
$$k\sqrt{\rho p} = k\exp(-Z/2)\, p^{(k+1)/2k} = \beta(Z, p) = \beta. \tag{8}$$

Now we change system (6) to the following form, excluding density from it on the basis of the equation of state and Eq. (7) and allowing for (8)

$$\frac{\partial u}{\partial t} \pm \frac{1}{\beta}\frac{\partial p}{\partial t} + (u \pm \alpha)\left(\frac{\partial u}{\partial x} \pm \frac{1}{\beta}\frac{\partial p}{\partial x}\right) = -f(u), \tag{9}$$

$$\frac{\partial Z}{\partial t} + u\frac{\partial Z}{\partial x} = 0.$$

The initial and boundary conditions are assigned with Eqs. (2)-(4) and the corresponding values of Z at one of the ends, depending on the direction of the velocity u.

We will use an explicit finite-difference scheme of approximation for the Eqs. of system (9) to obtain the numerical solution. The scheme satisfies the convergence conditions [6, Sec. 24]. We will use a grid that is uniform with respect to x and t and has cells $h = \Delta x$ and $\tau = \Delta t$, respectively. There are N + 1 nodal points for x, including the boundary points, where N = 1/h. The values of φ at the nodal point (i, n) will be designated by $\varphi_i^n = \varphi(x_i,\ t_n) = \varphi(ih,\ n\tau)$.

Since the velocity characteristics of the third equation of (9) will change sign during discharge, we should write two different schemes for its, corresponding to positive and negative directions of the velocity u. As a result we obtain

$$Z_i^{n+1} = Z_i^n - \frac{\tau}{h} u_i^n (Z_i^n - Z_{i-1}^n),$$

if $u_i^n \geqslant 0$ $(i = 1,\ 2,\ 3,\ \ldots,\ N-1,\ N;\ n = 0,\ 1,\ 2,\ \ldots)$, we prescribe $Z_0^{n+1} = \ln(T_0/p_0^{(k-1)/k})$,

$$Z_i^{n+1} = Z_i^n - \frac{\tau}{h} u_i^n (Z_{i+1}^n - Z_i^n),$$

if $u_i < 0$ $(i = 0,\ 1,\ 2,\ \ldots,\ N-1;\ n = 0,\ 1,\ 2,\ \ldots)$, we prescribe

$$Z_N^{n+1} = \ln(T_0/p_0^{(k-1)/k}). \tag{10}$$

In accordance with the convergence conditions, the derivatives with respect to the coordinate in the first equation of system (9) (top sign) are approximated by difference relations in reverse, while those in the first are approximated by difference relations in the forward direction. The corresponding finite-difference equations are written in the form

$$u_i^{n+1} + \frac{1}{\beta_i^n} p_i^{n+1} = u_i^n + \frac{1}{\beta_i^n} p_i^n - \frac{\tau}{h} (u_i^n + \alpha_i^n) \left[u_i^n - u_{i-1}^n + \frac{1}{\beta_i^n} (p_i^n - p_{i-1}^n) \right] - \tau f(u_i^n),$$

$$u_i^{n+1} - \frac{1}{\beta_i^n} p_i^{n+1} = u_i^n - \frac{1}{\beta_i^n} p_i^n - \frac{\tau}{h} (u_i^n - \alpha_i^n) \left[u_{i+1}^n - u_i^n - \frac{1}{\beta_i^n} (p_{i+1}^n - p_i^n) \right] - \tau f(u_i^n)$$

$$(i = 1,\ 2,\ 3,\ \ldots,\ N-1;\ n = 0,\ 1,\ 2,\ \ldots).$$

Solving this system relative to the quantities u_i^{n+1}, p_i^{n+1}, we obtain the following three-point (with respect to i) difference relations:

$$\begin{aligned} u_i^{n+1} &= u_i^n - \frac{\tau}{2h} u_i^n \left[u_{i+1}^n - u_i^n + \frac{1}{\beta_i^n} (-p_{i+1}^n + 2p_i^n - p_{i-1}^n) \right] - \\ &- \frac{\tau}{2h} \alpha_i^n \left[-u_i^n + 2u_i^n - u_{i-1}^n + \frac{1}{\beta_i^n} (p_{i+1}^n - p_{i-1}^n) \right] - \tau f(u_i^n), \\ p_i^{n+1} &= p_i^n - \frac{\tau}{2h} \beta_i^n u_i^n \left[u_{i+1}^n + 2u_i^n - u_i^n + \frac{1}{\beta_i^n} (p_{i+1}^n - p_{i-1}^n) \right] - \\ &- \frac{\tau}{2h} \beta_i^n \alpha_i^n \left[u_{i+1}^n - u_{i-1}^n + \frac{1}{\beta_i^n} (-p_{i+1}^n + 2p_i^n - p_{i-1}^n) \right] \\ &(i = 1,\ 2,\ 3,\ \ldots,\ N-1;\ n = 0,\ 1,\ 2,\ \ldots), \end{aligned} \tag{11}$$

which allows us to successively find u_i^{n+1}, p_i^{n+1} at all interior points of the interval (i = 1, 1, 2, 3, ..., N − 1). The values of the quantities at the points i = 0, i = N must be determined by using boundary conditions (3), (4).

We will find the values of u_0^{n+1}, p_0^{n+1} at the boundary point x = 0 (i = 0). For this, we write Eqs. (11) at the point i = 0. The right sides of the resulting equations contain the quantities u_{-1}^n, p_{-1}^n at the imaginary point i = −1 in the form of the complex $u_{-1}^n + p_{-1}^n/\beta_0^n$. Elimination of this complex from the two equations yields the relation

$$p_0^{n+1} - \beta_0^n u_0^{n+1} = \gamma_0^n, \tag{12}$$

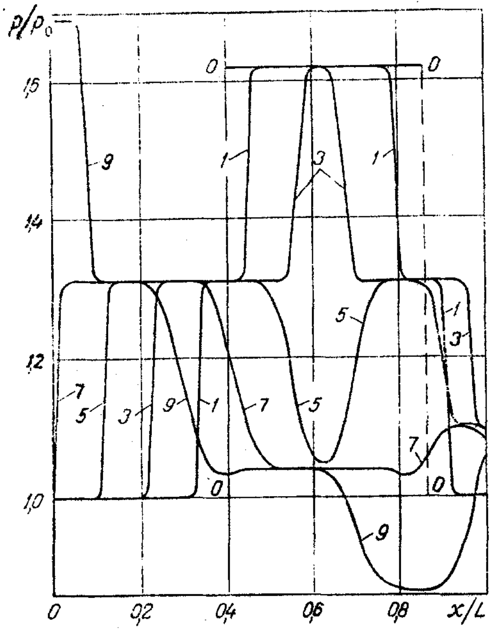

Fig. 1

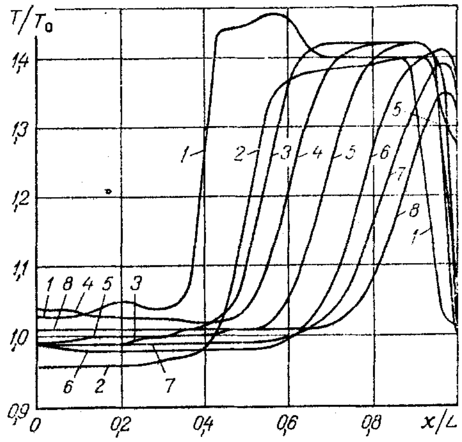

Fig. 2

Fig. 1. Pressure distribution along the cylinder for successive moments of time during the initial stage of the process.

Fig. 2. Temperature distribution along the cylinder for successive moments of time over 35 c.t.u.

where

$$\gamma_0^n = p_0^n - \beta_0^n u_0^n - \frac{\tau}{h}\beta_0^n(\alpha_0^n - u_0^n)\left[-u_0^n + u_1^n + \frac{1}{\beta_0^n}(p_0^n - p_1^n)\right] + \beta_0^n f(u_0^n).$$

The second equation for p_0^{n+1}, u_0^{n+1} will be written as follows on the basis of condition (4):

$$p_0^{n+1} + k\xi\rho_0^n \frac{(u_0^{n+1})^2}{2} = p^*. \qquad (13)$$

Excluding p_0^{n+1} from Eqs. (12), (13), we obtain a quadratic equation for u_0^{n+1}. The positive root of this equation has the form (the negative root has no meaning)

$$u_0^{n+1} = \frac{1}{k\xi\rho_0^n}\left(-\beta_0^n + \sqrt{(\beta_0^n)^2 + 2k\xi\rho_0^n(p^* - \gamma_0^n)}\right). \qquad (14)$$

The value of p_0^{n+1} is found from (12) by substituting the quantity u_0^{n+1} from (14) into it.

Similar reasoning allows us to use system (11), written for the point $i = N$, and boundary condition (3) to obtain the values of u_N^{n+1}, p_N^{n+1} at the boundary point $x = 1$ $(i = N)$:

$$u_N^{n+1} = \frac{1}{k\rho_N^n}\left(\beta_N^n + \sqrt{(\beta_N^n)^2 - 2k\rho_N^n(p_0^* - \gamma_N^n)}\right),$$
$$p_N^{n+1} + \beta_N^n u_N^{n+1} = \gamma_N^n, \qquad (15)$$

where

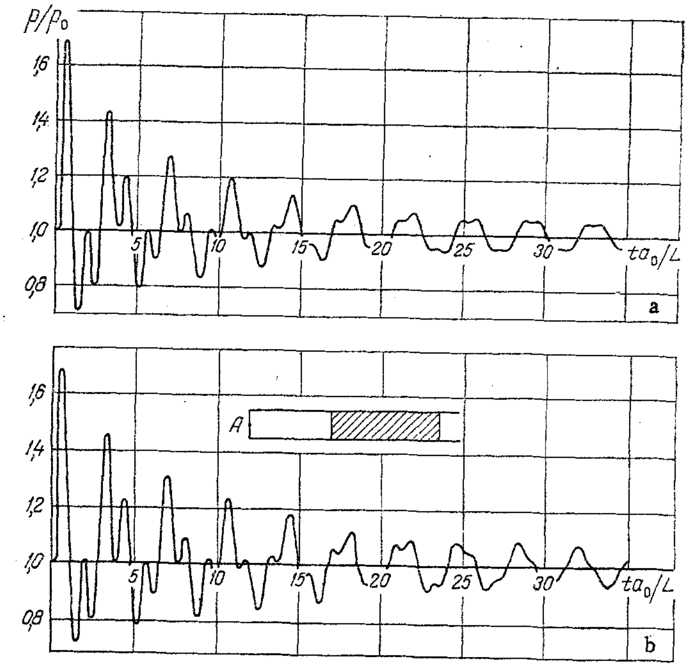

Fig. 3. Time dependence of pressure at point A of the cylinder (closed end). The heating zone is in the middle of the cylinder (hatched region): a) with constant motion of the gas in the cylinder at a velocity $u_0 = 0.03$) b) with the medium at rest in the unperturbed state.

where

$$\gamma_N^n = \beta_N^n u_N^n + p_N^n - \frac{\tau}{h}\beta_N^n (u_N^n + \alpha_N^n)\left[u_N^n - u_{N-1}^n - \frac{1}{\beta_N^n}(p_N^n - p_{N-1}^n)\right] - \tau\beta_N^n f(u_N^n).$$

Calculations were performed for the following numerical values of the quantities (in the equations and the initial and boundary conditions):

$$x_1 = 0.4;\ x_2 = 0.86;\ k = 1.4;\ u_0 = 0.03;\ p^* = 4;\ p_0 = p_0^* = T_0 = 1;\ p_1 = T_1 = 1.62;\ \rho_1 = \rho_0 = 1.$$

As in [1], the function $f(u_i^n)$ was determined from the following relation in our calculations

$$f(u_i^n) = 1.97\cdot 10^{-2}\,[1 + 1.38\,(\rho_i^n u_i^n)^{-0.237}](u_i^n)^2. \tag{16}$$

The parameter α in the convergence condition of problem (10)-(15) $\alpha\tau/h < 1$ (the Curant, Friedrichs, and Levy condition) is determined in the following manner as the highest value of the three characteristic velocities during discharge: $\alpha = u_1 + \sqrt{p_1/\rho_1} = 0.42 + \sqrt{1.62} = 1.69$. With this value of α, the convergence condition is written in the form $\tau < 0.6h$. All of the calculations were performed with $h = 0.01$ and $\tau = 0.005$. The pressure distribution in Fig. 1 was obtained with a greater accuracy with $h = 0.002$ and $\tau = 0.001$. The calculations were performed on an "M-4030" computer. The results were printed out in the form of graphs of the distribution of gasdynamic quantities (p, u, T) along x for successive moments of time, with a certain time interval (there was usually 10 curves per figure). Here, we used the graph-drawing program developed by S. I. Shabun.

Figure 1 shows the pressure distribution along the cylinder axis x for the initial stage of discharge. The five curves in this figure correspond to successive moments of time reckoned from the beginning of the process in accordance with the relation $t_n = n\Delta t$, where n = 0, 1, 3, 5, 7, and 9 and $\Delta t = 0.05$ characteristic time units (c.t.u.), i.e., 1 c.t.u. = L/a_0. The number 0 denotes the initial pressure distribution. Decay of the initial pressure jump is accompanied by the appearance and propagation of perturbations in the form of rarefaction waves and compression waves (weak shock waves). Rarefaction waves orginating from the boundary points of the initial pressure jump x_1 and x_2 move toward each other with an intensity half that of the initial pressure jump so that the "foot" of these waves is the "crest" of the compression waves (their intensity is also half the initial jump), which propagate in opposite directions from the points x_1 and x_2. The interaction of the rarefaction waves at the midpoint of the segment (x_1, x_2) (curve 3) results in a drop in pressure at this point and the beginning of propagation of new rarefaction waves (curve 5) in opposite directions relative to this point. The right-hand rarefaction wave meets the rarefaction wave moving counter to it at about the point x_2. The latter wave represents the reflection of the compression wave from the open end. The interaction of these waves leads to a pressure drop at the point x_2 to a value below the pressure in the environment (curve 7 on the right). The left half of the figure can be similarly explained. Figure 1 well illustrates the advantages of the method chosen here (compare it with Fig. 1a and c in [1]).

To explain the effect of through motion on the gasdynamic processes in the cylinder, we perform calculations embracing the time interval from the beginning of the process to the elapse of 35 c.t.u. The results were represented in the form of graphs of the pressure (and temperature) distribution, similar to those shown in Fig. 1 (only 10 curves per figure), for successive time intervals with $\Delta t = 0.25$ c.t.u. The calculations were performed for two variants: 1) without through motion $u_0 = 0$; 2) with through motion at a velocity $u_0 = 0.03$ in the unperturbed state. The resulting distributions can be used to find the time dependence of the gasdynamic quantities for any point of the cylinder. To this end, for a sequence of points on the axis t with the interval $\Delta t = 0.25$ c.t.u., we plotted values of p representing points of intersection of the corresponding curves with a vertical straight line passing through a chosen point on the cylinder axis.

Figure 2 shows the temperature distribution along x for successive moments of time. The initial temperature distribution is represented through zero. Curve 1 corresponds to the moment of time from the beginning of the process to 0.9 c.t.u. All of the other curves correspond to moment of time in c.t.u's which can be determined from the formula $t_n = (n - 1)\Delta t + 0.9$, where n is the number of the curve (n = 2, 3, ..., 8), $\Delta t = 4.5$ c.t.u. It is apparent that the contact surfaces are shifted to the right and after about 35 c.t.u. the left contact surface reaches the position of the right boundary of the inital temperature jump. This means that the heated medium is completely removed from the heating region.

Figure 3 shows the time dependence of the pressure at the closed end of the cylinder. It is apparent that constant motion does not introduce any substantial changes in the character of decay of the perturbations, and the curves are somewhat different from those for later times in the process only in structure. According to the curves of pressure distribution along x which were used to plot the curves in Fig. 3, the minimum and maximum pressures in the perturbations for the time ~35 c.t.u. are equal to 0.055 and 1.053 for a and 0.922 and 1.085 for b, respectively. The deviations of pressure from the normal value relative to the initial jump are: a — 8%; b — 13%.

NOTATION

x, coordinate along the cylinder axis; t, time; u, velocity; p, pressure; ρ, density; T, temperature; k, adiabatic exponent; a, speed of sound; x_1, x_2, boundary points of region of initial pressure and temperature jump; r_h, hydraulic radius. Indices: 0, quantities outside the perturbation region; 1, initial values of quantities in the pressure and temperature jump.

LITERATURE CITED

1. É. T. Bruk-Levinson, O. G. Martynenko, and E. A. Romashko, "Unidirectional nonsteady discharge of an instantaneously heated gas from a cylinder with different locations of the heating zone," Inzh.-Fiz. Zh., 43, No. 6, 1020-1027 (1982).
2. M. J. Levy and J. H. Potter, "Gas flow in a rarefaction wave tube," Nav. Eng. J., December (1964), pp. 941-950.
3. I. P. Ginzburg, Applied Hydrogasdynamics [in Russian], Leningrad State Univ. (1958).
4. S. K. Godunov, Equations of Mathematical Physics [in Russian], Nauka, Moscow (1971).
5. É. T. Bruk-Levinson and E. A. Romashko, "Unidirectional adiabatic discharge of an instantaneously heated gas from a cylinder into a medium with a finite pressure," in: Physics and Engineering of Aerothermooptical Methods of Controlling and Analyzing Laser Radiation [in Russian], ITMO Akad. Nauk Belorussian SSR, Minsk (1981), pp. 76-87.
6. S. K. Godunov and V. S. Ryaben'kii, Difference Methods [in Russian], Nauka, Moscow (1973).